Deep Learning accelerator for model-based reconstructions of tomographic images

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LEARNING AND OPTIMIZATION FOR IMAGING IN BOLOGNA
• LOIBO •

OASIS
Optimization Algorithms and Software for Inverse problemS

MiVA

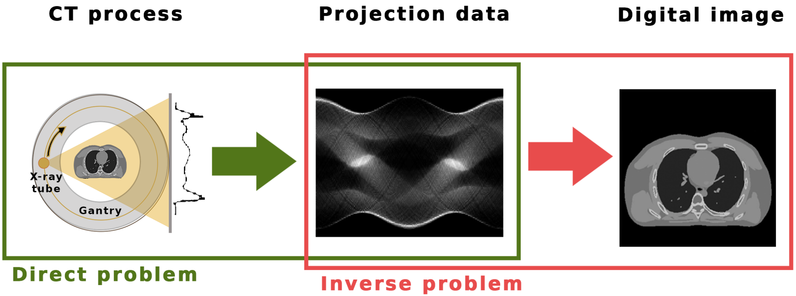
1. Preliminaries on X-ray CT image reconstruction

2. Deep Guess acceleration strategy

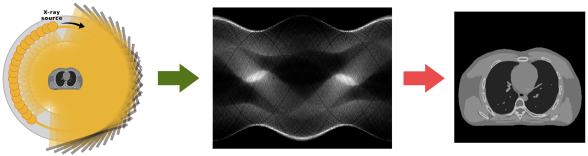
3. Results

4. Conclusion

The aim of CT is to compute the digital image of the human body from the projection data acquired during the CT body scan.



Tomographic inverse problem with model-based approach



CT image reconstruction means finding $x \in \mathbb{R}^m$ such that:

$$Ax + e = b$$

- $A \in \mathbb{R}^{m \times n}$ is the discretization of the forward projection,
- $b \in \mathbb{R}^n$ stores data,
- e represents noise

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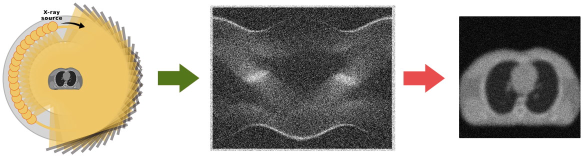

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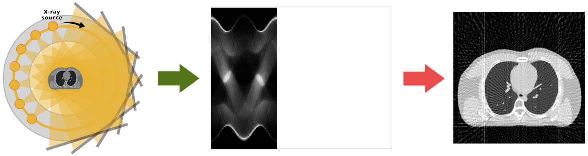

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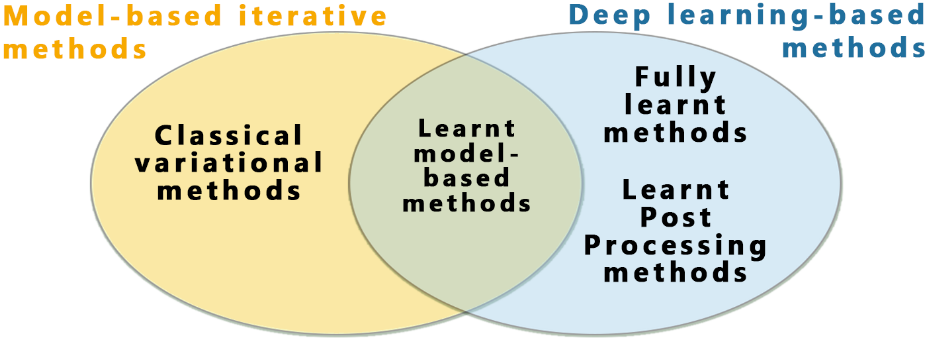
The linear system

- becomes underdetermined in case of sparse-view CT,
- is ill-posed, due to the Lambert-Beer's law.

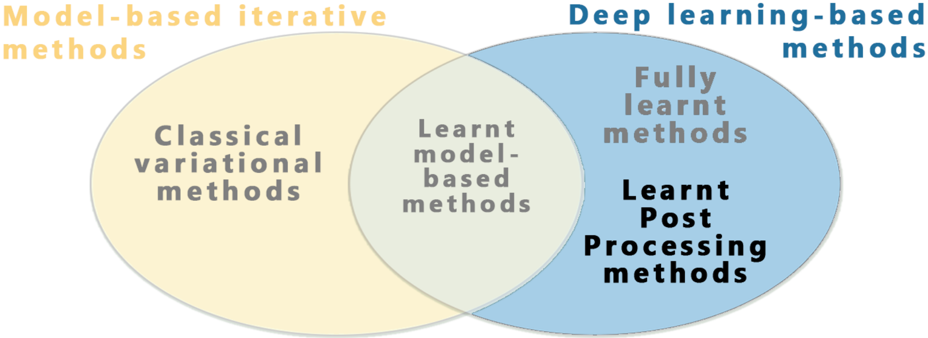
Hence *iterative methods* are used, solving sparsity-promoting model-based formulations

$$min_{x \in \Omega} \; \mathcal{F}(x; b, A) + \lambda \mathcal{R}(x)$$

Approaches


Model-based iterative methods
Deep learning-based methods
Classical variational methods
Learnt model-based methods
Fully learnt methods
Learnt Post Processing methods

Approaches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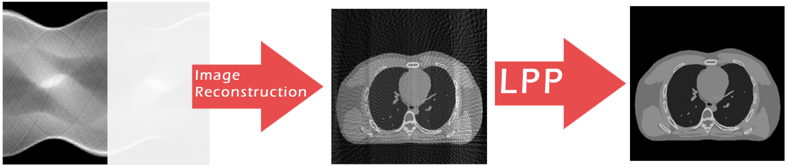
Learned Post-Processing approach



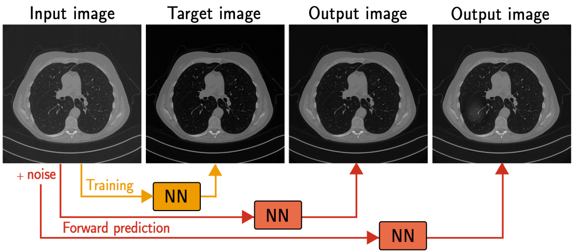
- Fast execution: coarse reconstruction and network inference;
- Loss of (mathematical) interpretability of the final solution.

DL for inverse problems?

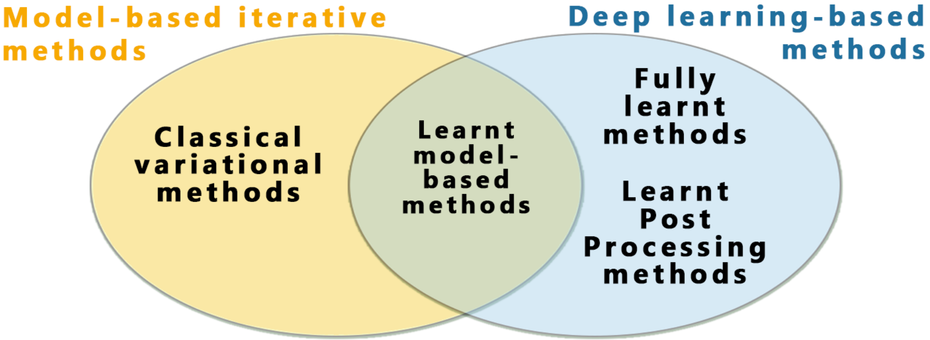
"In the sparse-view CT literature using CNNs, there is not clear evidence that an associated inverse problem is being solved."

Sidky Emil, Pan Xiaochuan, et al. " Do CNNs solve the CT inverse problem?", IEEE Transactions on Biomedical Engineering (2020).

Neural Networks (alone) are not stable operators[1], hence theoretical understanding and analysis of DL-based image processing are necessary[2].



[1] S. Bhadra et al., "On hallucinations in tomographic image reconstruction." IEEE transactions on medical imaging, (2021).
D. Evangelista et al., "Ambiguity in solving imaging inverse problems with deep-learning-based operators." Journal of Imaging (2023).
[2] N. M. Gottschling, V. Antun, A. C. Hansen, B. Adcock, "The Troublesome Kernel: On Hallucinations, No Free Lunches, and the Accuracy-Stability Tradeoff in Inverse Problems." SIAM Review, (2025).
D. Evangelista et al., "To be or not to be stable, that is the question: understanding neural networks for inverse problems." SIAM Journal on Scientific Computing (2025).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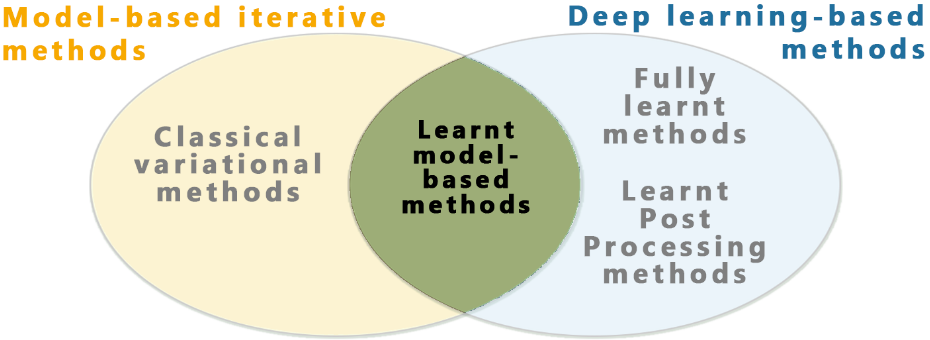
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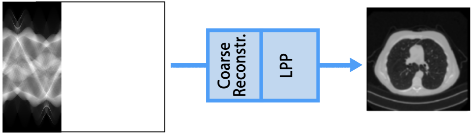
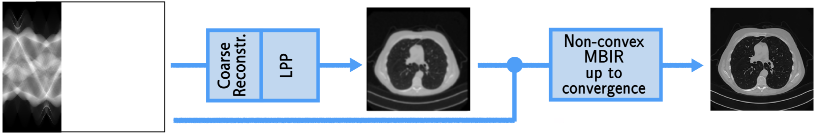
Deep Guess for non-convex optimization[3]



[3] E. Loli Piccolomini, D. Evangelista, E. Morotti, "Deep Guess acceleration for explainable image reconstruction in sparse-view CT", Computerized Medical Imaging and Graphics,(2025).

Deep Guess for non-convex optimization[3]



- High data-consistency: the model-based solver (CP) improves the NN image;

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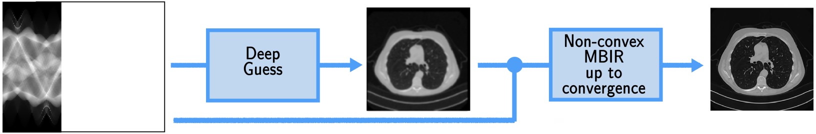

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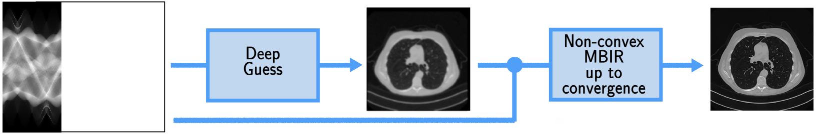


- High data-consistency: the model-based solver (CP) improves the NN image;
- Reduced risk of bad local minima: with a better initial guess, the likelihood of the solver falling into a bad image is significantly decreased;
- Fast execution: the iterative solver can start close to its solution, reducing the number of needed iterations.

[3] E. Loli Piccolomini, D. Evangelista, E. Morotti, "Deep Guess acceleration for explainable image reconstruction in sparse-view CT", Computerized Medical Imaging and Graphics,(2025).

Non-convex imaging model for sparse data

To force sparsity in the image gradient domain, we use:

$$\min_{\mathbf{x}\geq 0}\left(||A\mathbf{x}-b||_2^2+\lambda\ TpV(\mathbf{x})\right) \tag{1}$$

where the Total p-norm Variation ($p<1$) reads:

$$TpV(\mathbf{x}) := ||D\mathbf{x}||_p^p = \sum_{i=1}^{n}\left(\sqrt{(\nabla_h\mathbf{x})_i^2+(\nabla_v\mathbf{x})_i^2}\right)^p. \tag{2}$$

Remarks:

- the ℓ_0 quasi-norm is the best sparsifying function, but it makes the problem computationally difficult;
- the ℓ_p with $0\leq p\leq 1$ is a non-convex good approximation of ℓ_0.

The TpV Chambolle-Pock algorithm

Exploiting the reweighting strategy[4], the CP has been applied to a reformulation of the *TpV* non-convex minimization into a convex weighted TV problem[5]. Indeed, we can alter the objective function in (1) as:

$$\min_{x\geq 0} \left(|A\mathbf{x} - b||_2^2 + \lambda \mid\mid \boldsymbol{w} \odot Dx \mid\mid_1 \right) \tag{3}$$

where the element-wise product $\odot$ uses weights $\boldsymbol{w} \in \mathbb{R}^n$ defined as:

$$\boldsymbol{w} = \left(\frac{\sqrt{\eta^2 + |\nabla \mathbf{x}|^2}}{\eta} \right)^{p-1}.$$

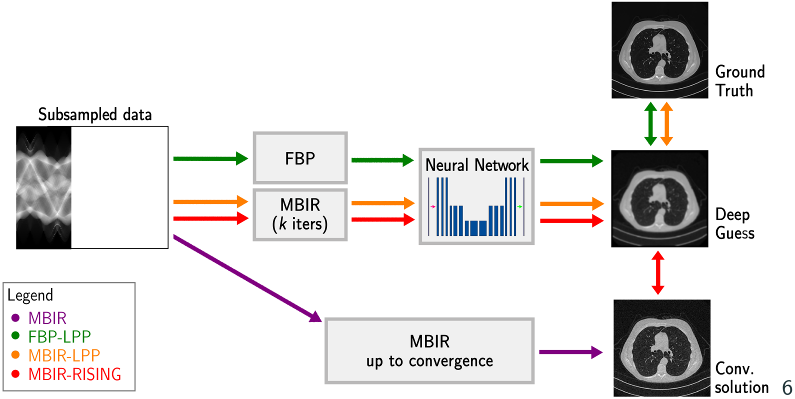
The weights are iteratively updated.

[4] Candes et al., "Enhancing sparsity by reweighted l1 minimization", (2008)

[5] Sidky et al., "Constrained TpV minimization for enhanced exploitation of gradient sparsity: Application to CT image reconstruction", (2014)

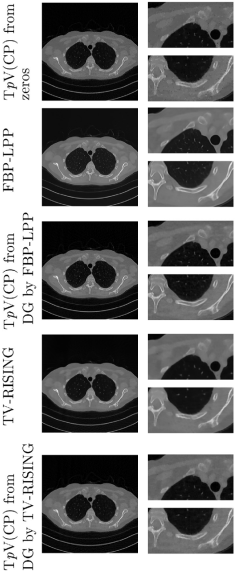
Implementations of Deep Guess step

We designed three Deep Guess schemes, based on the ResUNet architecture:



[6] D. Evangelista, E. Morotti, and E. Loli Piccolomini. "RISING: A new framework for model-based few-view CT image reconstruction with DL." Computer. Medical Imaging and Graphics (2023).

Results on a real medical image



Experimental set-up:

- 512x512 Mayo images;
- 60 scans in 180 degrees;
- noise level $nl = 0.001$;
- $p = 0.2$, $k = 15$;

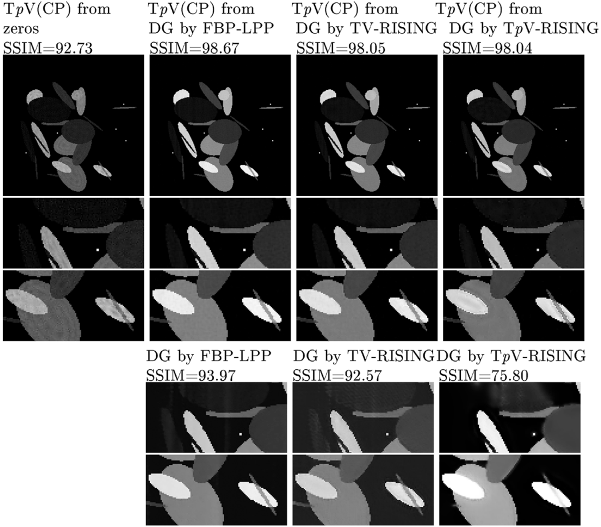
Values of the quantitative metrics, computed on the Mayo Clinic test image reconstructions for the $\mathcal{G}_{180,60}$ protocol, with different initial guesses $\boldsymbol{x}^{(0)}$ for the CP and for $p = 0.5$ and $p = 0.2$, and with state-of-the-art method.

	$\boldsymbol{x}^{(0)}$			Output		
		SSIM	RE	SSIM	RE	iters
TpV(CP) with $p = 0.5$	zeros	–	–	78.08	0.1039	500
	FBP	30.56	0.2955	77.01	0.1076	400
	TV ($K = 15$)	67.18	0.1906	75.36	0.1147	400
	DG by FBP-LPP	**80.74**	**0.0876**	**83.16**	**0.0792**	12
	DG by TV-RISING	76.99	0.1059	78.79	0.0989	19
TpV(CP) with $p = 0.2$	zeros	–	–	77.03	0.1071	500
	FBP	30.56	0.2955	76.35	0.1102	400
	TV ($K = 15$)	67.18	0.1906	76.43	0.1093	400
	DG by FBP-LPP	**80.74**	**0.0876**	**82.75**	**0.0794**	18
	DG by TV-RISING	76.99	0.1059	79.65	0.0958	36
NETT (Li et al., 2020)	–	–	–	61.12	0.1576	300
LPD (Adler and Öktem, 2018)	–	–	–	79.53	0.1033	10

Results on synthetic images

Experimental set-up:

- 256x256 COULE images;
- 180 scans in 180 degrees;
- noise level $nl = 0.01$;
- $p = 0.5$, $k = 10$;
- $||x^{(k)} - x^{(k+1)}||_2 < 10^{-3}$. (Conv. of T$p$V-CP in 192 iters.)



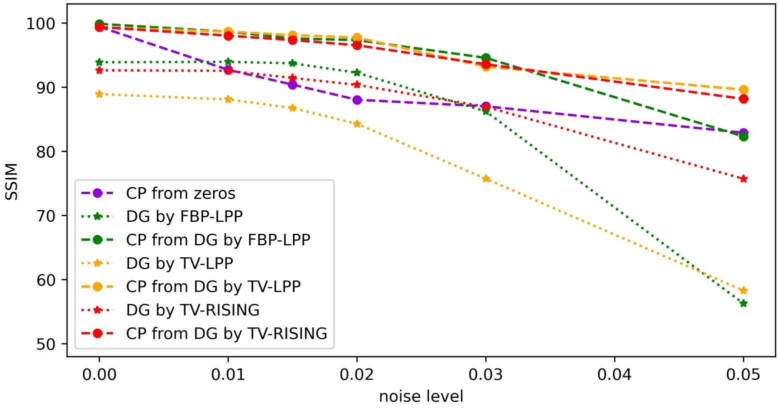
Ground Truth: 

Mean values (standard deviations) of the quantitative metrics, computed on the COULE test images. The first two columns present the metric values for the computed image $\boldsymbol{x}^{(0)}$ (DG step); on the right, the metrics for the final reconstructed image by the DG accelerated TpV(CP) are shown.

	$\boldsymbol{x}^{(0)}$		Output		
	SSIM	RE	SSIM	RE	iters
zeros	–	–	88.70 (3.37)	0.0851 (0.0073)	200
DG by FBP-LPP	90.02 (4.19)	0.0642 (0.0163)	96.20 (2.34)	0.0321 (0.0057)	45-55
DG by TV-LPP	87.45 (3.28)	0.0527 (0.0074)	97.32 (1.03)	0.0290 (0.0025)	20-32
DG by TV-RISING	90.79 (2.60)	0.0619 (0.0105)	96.17 (2.49)	0.0341 (0.0047)	47-55
DG by TpV-LPP	93.80 (2.07)	0.1372 (0.0517)	94.54 (4.23)	0.0530 (0.0120)	75-80
DG by TpV-RISING	83.78 (10.84)	0.1434 (0.0515)	93.01 (4.49)	0.0538 (0.0142)	73-85

- TpV-CP without DG performs the worst among all solvers;
- TpV-CP with DG consistently improves the reconstructed image, while also reducing the standard deviation values (in most cases);
- TV-LPP and TV-RISING achieve moderate SSIM scores, but they serve as the best deep guesses;
- The number of iterations required is significantly reduced when using DG.

Stability results on a synthetic image



While the networks' predictions deteriorate, the TpV-CP solutions (above all with TV-LPP and TV-RISING deep guesses) preserve good qualities!

Our ongoing/next steps

Many considerable research opportunities still exist in the domain of Learned Model-Based Methods for sparse tomographic imaging:

- theoretical understanding and analysis of DL-based processing (and stability);
- test on real clinical data provided by our industrial collaborations;
- design of new mathematically-grounded hybrid frameworks for clinical usage.

1. D. Evangelista, E. Morotti, and E. Loli Piccolomini.
RISING: A new framework for model-based few-view CT image reconstruction with deep learning.
Computerized Medical Imaging and Graphics, 2023.
2. D. Evangelista, E. Morotti, E. Loli Piccolomini, and J. Nagy.
To be or not to be stable, that is the question: understanding neural networks for inverse problems.
SIAM Journal on Scientific Computing (2025)
3. E. Loli Piccolomini, D. Evangelista, E. Morotti.
Deep Guess acceleration for explainable image reconstruction in sparse-view CT.
Computerized Medical Imaging and Graphics, (2025)
4. E. Morotti.
An incremental algorithm for non-convex AI-enhanced medical image processing.
arXiv preprint, 2025.

Thank you!